Joint Coverage and Backhaul Self-Optimization in Emerging Relay Enhanced Heterogeneous Networks

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***Abstract*—This paper presents a novel framework for joint self-optimization of backhaul as well as coverage links spectral efficiency in relay enhanced heterogeneous networks. Considering a realistic heterogeneous network deployment, where some cells contain Relay Station (RS), while others do not, we develop an analytical framework for self-optimisation of macrocell Base Station (BS) antenna tilts. Our framework exploits a unique system level perspective to enable dynamic maximization of system-wide spectral efficiency of the BS-RS backhaul links as well as that of the BS-user coverage links. A distributed and practical self-organising solution is obtained by decomposing the large scale system-wide optimization problem into local small scale optimization problems, by mimicking the operational principles of self-organisation in biological systems. The local problems are non-convex but have very small scale and can be solved via appropriate numerical methods, such as sequential quadratic programming. The performance of developed solution is evaluated through extensive system level simulations for LTE-A type networks and compared against conventional tilting benchmarks. Numerical results show that up to 50% gain in average spectral efficiency is achievable through the proposed solution depending on users geographical distributions.**

I. Introduction

While the latest 3GPP releases (11 and 12) are looking at exploiting RSs further to meet the stringent capacity, QoS and energy efficiency demands from emerging cellular systems such LTE-A [1], the full potential of RS remains thwarted by two key challenging issues. The first challenge is to overcome the spectrum reuse inefficiency caused by the extra spectrum required for BS-RS *backhaul* link. The need for this extra spectrum severely limits RS potential of system-wide capacity enhancement in cellular systems. Therefore, it is highly desirable to optimise the Spectral Efficiency (SE) of backhaul links. Secondly, the BS infrastructure that has to support a RS based enhancement, should have self organizing capabilities to accommodate on-the-run deployment of the RSs. In other words, BSs should be able to autonomously update their operational parameters to accommodate the impromptu advent, departure or location change of RSs in their vicinity. Only such self organising capability can ensure the much needed OPEX and CAPEX saving in emerging cellular systems [2].

Although, exhaustive research efforts have been channeled into developing myriad of physical layer [3], [4], MAC layer [5]–[7] and network layer [8], [9] solutions to counteract the spectrum reuse inefficiency caused by the backhaul links of RSs, remarkably very less attention has been paid towards the solutions that can be harnessed by applying self-organising concepts from a system level perspective. More specifically, work on optimisation of the RSs' backhaul links by self-organising BSs radio access parameters such as antenna tilts, is scarce in literature [2]. Although, a significant number of works have embarked on tilt optimisation for coverage and capacity enhancement in macro cellular systems [10]–[13][1], to the best of the authors' knowledge, none of these works provides an antenna tilt self-optimising solution for relay enhanced heterogeneous cellular systems, while jointly taking into account both backhaul and coverage links. Authors in [14] introduced the concept of SE enhancement on the backhaul link through BS antenna tilt adaptation for the first time. However, the analysis is limited to a highly symmetric scenario, where all the cells are assumed to contain strictly one RS in each cell. Therefore, the solution in [14] does not take into account a realistic scenario where some cells might not contain RSs. This paper on the other hand, presents a novel solution for self optimisation of tilts in a realistic heterogenous deployment of relay enhanced cellular systems. By taking into account RS locations and statistics of user demography, we jointly maximise the SE of both backhaul and coverage links through optimisation of system-wide antenna tilts in a distributed and self-organized fashion.

The rest of the paper is organised as follows. In Section II we present the system model, the general assumptions and the problem formulation. In order to achieve a Self Organization (SO) solution, in Section III we propose a way to decompose the system-wide problem into local subproblems, as inspired by SO systems in nature. The solution methodology for the local sub-problems is also presented in this section. Section IV presents numerical and system level simulation results to demonstrate the gain achievable by the proposed solution. The conclusions are drawn in Section V.

II. General Assumptions and System Model

Assumptions and Nomenclature: We assume a frequency reuse of one with interference limited scenario. BSs and RSs are multiplexed in time (or frequency) such that there is no

[1]A detailed survey of works on tilt optimization can be found in our previous work in [2].

cross interference among the backhaul (BS-RS) and coverage (BS-user and RS-user) links. It is assumed that all user devices and RSs have omnidirectional antennas with 0dB gain. We use SE in b/s/Hz as an optimisation metric and we define it as the long term average bandwidth normalised throughput on a link given by $\log_2(1 + SIR)$, where SIR stands for Signal to Interference Ratio. Due to the geometrical context of the following analysis, by referring to BS, RS and users we mean the location of their antennas unless specified otherwise. Symbol *tilde* e.g. $\tilde{x}$ is used to denote optimal value of variable x and symbol *hat* e.g. $\hat{x}$ is used to denote an approximation of a variable x.

System Model: We consider the downlink scenario of a sectorised multi cellular network as shown in Figure 1. Each BS has three cells (sectors) and each cell has at most one RS station placed at an arbitrary location, to cover random hotspots of users. Let $\mathcal{B}$ denote the set of points corresponding to the transmission antenna location of all BS cells, $\mathcal{R}$ the set of points representing the locations of the RSs antennas in the system and $\mathcal{U}$ the set of points representing the antennas of all the user devices randomly located in the system. The geometric SIR on the backhaul link of a RS located at point $r \in \mathcal{R}$ associated with b^{th} cell, can be written as:

$$\gamma_r^b = \frac{P^b G_r^b G_r \alpha \left(d_r^b\right)^{-\beta} \delta_r^b}{\sum_{\forall \acute{b} \in \mathcal{B} \backslash b} \left(P^{\acute{b}} G_r^{\acute{b}} G_r \alpha \left(d_r^{\acute{b}}\right)^{-\beta} \delta_r^{\acute{b}}\right)} \quad b, \acute{b} \in \mathcal{B} \qquad (1)$$

where P^b is the transmission power of the b^{th} cell, d_r^b and $d_r^{\acute{b}}$ are the distances between the b and $\acute{b}$ transmitting cell antenna locations and receiving RS antenna location r. α and β are the pathloss model coefficient and exponent, respectively. δ_r^b and $\delta_r^{\acute{b}}$ are shadowing coefficients that represent shadowing faced by a signal at location r while being received from the b^{th} and $\acute{b}^{th}$ BS antennas, respectively The operator '$\backslash$' in $\mathcal{B}\backslash b$ means all elements of $\mathcal{B}$ excluding b. G_r^b and $G_r^{\acute{b}}$ are the antenna gains perceived at RS r, from BS b and $\acute{b}$, respectively. For 3GPP LTE and LTE-A the three dimensional antenna pattern can be modelled as proposed in [15], and with the simplifications introduced in [14]. Using the geometry in Figure 1, the perceived antenna gain from a b^{th} BS, at location r, of a RS can be written in dBs as follows:

$$G_r^b = 10^{-1.2\left(\lambda_v \left(\frac{\psi_r^b - \psi_{tilt}^b}{B_v}\right)^2 + \lambda_h \left(\frac{\phi_r^b - \phi_a^b}{B_h}\right)^2\right)} \qquad (2)$$

where ψ_r^b is the vertical angle at the b^{th} BS, in degrees from reference axis (horizon) to the r^{th} RS. Here ψ_{tilt}^b is the tilt angle of the b^{th} cell, as shown in Figure 1. The ϕ_a^b is the angle of the azimuth orientation of the antenna with respect to the horizontal reference axis, i.e. positive x-axis. ϕ_r^b is the angle of location r of the RS from the horizontal reference axis, at BS b. Subscripts h, a and v denote horizontal, azimuth and vertical, respectively. Thus B_h and B_v represent the horizontal and vertical beamwidths of the BS antenna, respectively, and λ_h and λ_v represent the weighting factors for the horizontal and vertical beam patterns of the antenna in the 3D antenna model [15], respectively.

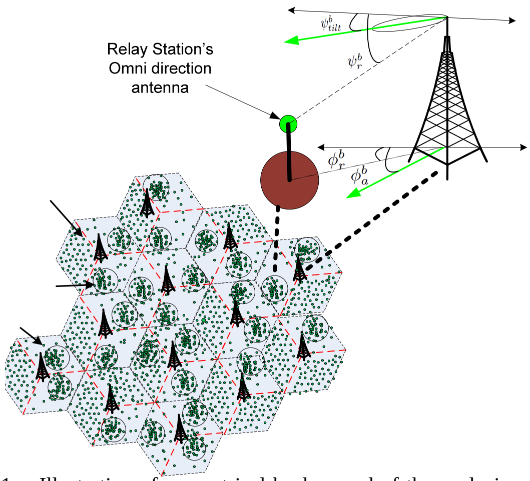

Fig. 1. Illustration of geometrical background of the analysis.

We assume that all the BSs transmit with the same power and all RS antennas have unity gain i.e. $G_r = 1$. For the sake of simplicity, we use the following substitutions:

$$c_k^b = \frac{B_v^2 \lambda_h}{\lambda_v}\left(\frac{\phi_r^b - \phi_a^b}{B_h}\right)^2 ; \quad c_k^{\acute{b}} = \frac{B_v^2 \lambda_h}{\lambda_v}\left(\frac{\phi_r^{\acute{b}} - \phi_a^{\acute{b}}}{B_h}\right)^2 \qquad (3)$$

$$h_r^b = \delta_r^b \alpha \left(d_r^b\right)^{-\beta} \quad ; \quad h_r^{\acute{b}} = \delta_r^{\acute{b}} \alpha \left(d_r^{\acute{b}}\right)^{-\beta} ; \quad \mu = \frac{-1.2\lambda_v}{B_v^2} \qquad (4)$$

Using the substitutions in (1) $-$(4) the SIR on the backhaul link of the r^{th} RS can be determined as:

$$\gamma_r^b = \frac{h_r^b 10^{\mu\left(\left(\psi_r^b - \psi_{tilt}^b\right)^2 + c_r^b\right)}}{\sum_{\forall b \in \mathcal{B} \backslash \acute{b}} \left(h_r^{\acute{b}} 10^{\mu\left(\left(\psi_r^{\acute{b}} - \psi_{tilt}^{\acute{b}}\right)^2 + c_r^{\acute{b}}\right)}\right)} \qquad (5)$$

Note that γ_r^b is a function of $\boldsymbol{\psi}_{tilt}^B = \left[\psi_{tilt}^1, \psi_{tilt}^2, \psi_{tilt}^3 ... \psi_{tilt}^B\right]$, where $B = |\mathcal{B}|$, but for the sake of simplicity, we will show this dependency only where necessary. Following the same steps as above, the SIR for the BS-user link can be written as:

$$\gamma_u^b = \frac{h_u^b 10^{\mu\left(\left(\psi_u^b - \psi_{tilt}^b\right)^2 + c_u^b\right)}}{\sum_{\forall \acute{b} \in \mathcal{B} \backslash b} \left(h_u^{\acute{b}} 10^{\mu\left(\left(\psi_u^{\acute{b}} - \psi_{tilt}^{\acute{b}}\right)^2 + c_u^{\acute{b}}\right)}\right)} \qquad (6)$$

III. TILT OPTIMIZATION FRAMEWORK

A. Problem Formulation

To incorporate different types of relay stations (coverage or capacity enhancing or RS with different radii in a heterogenous network) and users (prime or regular etc), we model our problem i.e joint optimisation of both backhaul and coverage links through antenna tilt optimization as in (7), where $0 < w_r \leq 1$ is a weight factor that varies over a fixed range of 0-1. These weights can be set to model the significance of each RS depending on statistics of the number and activity levels of users it serves. These weights can also be used to reflect if a RS has been deployed for coverage extension and therefore might have low load backhaul that needs to be assigned a lower weight. In case the RS has been deployed for capacity

$$\max_{\boldsymbol{\psi}_{tilt}^{B}} \left(\frac{1}{W_r} \sum_{\forall r \in \mathcal{R}} w_r \log_2 \left(1 + \gamma_r^b \left(\boldsymbol{\psi}_{tilt}^{B}\right)\right) + \frac{1}{\acute{A}_u} \sum_{\forall u \in \mathcal{U} \setminus \acute{\mathcal{U}}} a_u \log_2 \left(1 + \gamma_u^b \left(\boldsymbol{\psi}_{tilt}^{B}\right)\right) \right) \tag{7}$$

extension at a hotspot it might have a heavily loaded backhaul that needs to be assigned proportionally higher w_r. Where, $W_r = \sum_{\forall r \in \mathcal{R}} w_r$. In a simple example, w_r can be calculated as: $w_r = \frac{\sum_{\forall u \in \mathcal{U}_b^r} a_u}{\sum_{\forall u \in \mathcal{U}_b} a_u}$ $, 0 < a_u \leq 1$ .Where a_u represents u^{th} user activity level. $\mathcal{U}_b$ is set of users in the b^{th} BS cell and $\mathcal{U}_b^r$ is set of users in the r^{th} RS cell within b^{th} BS cell. Referring back to (7), $\acute{\mathcal{U}}$ is the set of users served by the RSs such that $\acute{\mathcal{U}} \subset \mathcal{U}$ and thus users served directly by the BS are given by set $\mathcal{U} \setminus \acute{\mathcal{U}}$ and $\acute{A}_u = \sum_{\forall u \in \mathcal{U} \setminus \acute{\mathcal{U}}} a_u$. Note that (7) is a nonlinear multi-variable optimisation problem. Its solution would require global cooperation among all cells in the system, which would make it not distributed and consequently not in line with the basic idea of online local self-organisation [2], [16]. Furthermore, as we will see in subsequent sections, the objective function in (7) is non-convex and characterised by a large number of the optimisation variables, i.e. $\boldsymbol{\psi}_{tilt}^{B} = \left[\psi_{tilt}^1, \psi_{tilt}^2, \psi_{tilt}^3 ... \psi_{tilt}^B\right]$, meaning that we are dealing with a large scale optimisation problem. Therefore, numerical or exhaustive search based heuristics are also not a practically feasible approach either. In the following section we present a novel biologically inspired approach to solve this problem in order to develop a pragmatic distributed self-organising solution.

B. Designing a Self Organising Solution

In nature many systems can be observed to exhibit self-organising behaviours. A detailed discussion on nature inspired SON design can be found in our earlier work in [2], and [16]. Here, it would suffice to say that for a self-organising solution, instead of targeting for a system-wide and globally optimal solution, which may be too complex to allow localised self organising implementation, we can opt for a sub-optimal approach. This rational is supported by the theory of self-organisation in biological systems, which rely on decomposing the global problem into local sub-problems. These sub-problems can be solved at local level, by requiring interactions only among local entities of system [16]. This approach is shown to achieve the original system wide objectives closely, while at the same time allowing localised self-organising behaviour [2], [16], characterised by autonomous capabilities and reduced complexity, implementation cost and signalling overheads. This design principle of self-organisation can be applied to our problem in (7). For that we need to: 1) find an alternative approximate representation of the problem in (7); 2) decompose that approximate problem into easily solvable local problems, whose solution would only require local coordination among neighbouring cells; and finally 3) determine the solution of those local subproblems. In the following three subsections we follow these three steps to achieve a self organising solution for problem in (7).

C. Simplifying the Problem to Achieve Decomposability

We present the following theorem that paves the way to determine a simpler and decomposable representation of (7).

Theorem 1. *If the tilt value for a given cell satisfies the condition:*

$$\sum_{u=1}^{|\mathcal{U}_b|} a_u \left(\left(\psi_u^b - \tilde{\psi}_{tilt}^b \right) \frac{\tilde{\gamma}_u^b}{1 + \tilde{\gamma}_u^b} \right) = 0 \tag{8}$$

it will yield greater or equal weighted average spectral efficiency on BS-user links than that obtained with any other value of tilt, for the same tilt angles of neighbouring cells. Note that γ_u^b here is function of antenna tilt of the b^{th} cell only, as rest of the antenna tilts are fixed. Summation in (8) *sums over all user in the cell.*

Proof: The SE in (7) *is a twice differentiable function of the tilt, therefore, the proof of theorem 1 can be easily obtained by finding the optimality conditions through the first derivative of the sum of the SE at all user locations, and by means of the second derivative test confirming that this condition provides a maximum point. Details are omitted for space limit.* ■

As a result, Theorem 1 provides a method to calculate the optimal tilts that maximize the BS-user link SE, in cells without RS. The following corollary can be directly deduced from Theorem 1:

Corollary 1. *For given tilt angles of neighbouring cells, the optimal tilt angle $\tilde{\psi}_{tilt}^b$ of cell b is the tilt angle that optimizes the SE at any point p. Where p belongs to a set of points $\mathcal{P}^b$ in that cell such that $\mathcal{P}^b = \{p|, d(p \leftrightarrow b) = d^b\}$, and where $d^b = \left(H^b - H^p\right) / \tan(\tilde{\psi}_{tilt}^b)$. H^b and H^p are the heights of the b^{th} cell antenna and point p, respectively. $d(p \leftrightarrow b)$ denotes the distance between the location of cell b antenna and the user location p.*

Proof: This corollary follows from Theorem 1, from the fact that the optimal tilt angle $\tilde{\psi}_{tilt}^b$ given by Theorem 1 can be transformed into a set of certain points $\mathcal{P}^b$, which lie at distance d^b from the cell antenna b. This is illustrated in figure 2. ■

Using theorem 1 and its corollary, the user distribution in each cell can be represented by a single focal point, (see Figure 2) for the tilt optimization process for an arbitrary user distribution and user activity profile. For ease of discussion, we refer to this focal point as Center of Gravity (CG) of a cell for its given user distribution and user activity profile. If the collection of all such CGs in the system is given by the set $\mathcal{V}$ it can be defined as $\mathcal{V} = \bigcup_{\forall b \in \mathcal{B}} \tilde{p}^b$ where $\tilde{p}^b \in \mathcal{P}^b$, by using this definition of $\mathcal{V}$ in conjunction with its corollary,

$$\frac{1}{\acute{A}_u}\sum_{\forall u\in\mathcal{U}\setminus\acute{\mathcal{U}}} a_u \log_2\left(1+\gamma_u^b\left(\boldsymbol{\psi}_{tilt}^{B}\right)\right) \equiv \sum_{\forall v\in\acute{\mathcal{V}}} \log_2(1+\gamma_v^b\left(\boldsymbol{\psi}_{tilt}^{B}\right) + \sum_{\forall v\in\check{\mathcal{V}}} \log_2(1+\gamma_v^b\left(\boldsymbol{\psi}_{tilt}^{B}\right)) \tag{9}$$

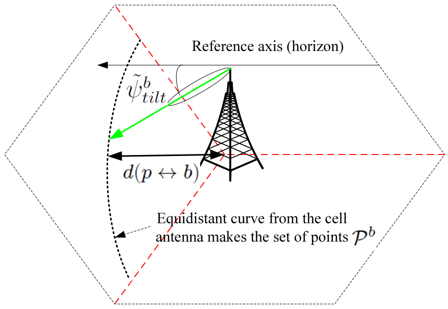

Fig. 2. The optimal tilt $\hat{\psi}^b_{tilt}$ cab be mapped to a set of points equidistant distant from the BS.Any of the points on this equidistant curve can be taken as the Center of Gravity (CG) of the user geographical distribution for which the optimal tilt angle is calculated.

the 2^{nd} summation of the right hand side of the optimization problem in (7) can be written as (9). Where $\acute{\mathcal{V}}$ and $\check{\mathcal{V}}$ are sets of CGs representing BS associated users in cells with RS and without RS respectively, such that $\mathcal{V} = \left\{\acute{\mathcal{V}} \cup \check{\mathcal{V}}\right\}$. To further simplify our optimization problem in (7) we propose the following generic method to determine a single point s_b that can represent the effective CG in each cell for the purpose of tilt optimization, including the cells that contain coverage or capacity enhancing RSs:

$$s_b = \begin{cases} r_b, & \text{if } |\mathcal{U}_b^r| > 0 \;\&\; w_r \geq \frac{\sum_{\forall u\in\mathcal{U}_b^b} a_u}{\sum_{\forall u\in\mathcal{U}_b} a_u}, \text{where } r_b \in \mathcal{R} \\ \acute{v}_b, & \text{if } |\mathcal{U}_b^r| > 0 \;\&\; w_r < \frac{\sum_{\forall u\in\mathcal{U}_b^b} a_u}{\sum_{\forall u\in\mathcal{U}_b} a_u}, \text{where } \acute{v} \in \acute{\mathcal{V}} \\ \check{v}_b, & \text{otherwise, where } \check{v} \in \check{\mathcal{V}} \end{cases} \tag{10}$$

where $\mathcal{U}_b^b$ denotes the set of users in the b^{th} cell that are directly associated with the BS. Thus case 1 of (10) refers to the scenario where the RS is serving the majority of users and thus is expected to have a capacity limited backhaul link that must be considered in the tilt optimization process. This case is applicable to capacity enhancing RS installed at hotspots in a cell. The second case of (10) represents the cells where the main purpose of the RS is coverage extension. The backhaul of such RS is not expected to be capacity limited and therefore does not have to be considered directly in the tilt optimization problem. In this case the CG of the respective cell will be determined by the users associated directly with the BS. The third case of equation (10) represents the cells with no RSs. Now if we define $\mathcal{S}$ as set of all points s_b in the system such that $|\mathcal{S}| = |\mathcal{B}|$, based on arguments presented above through (7)$-$(10), the problem in (7) can be written as:

$$\max_{\boldsymbol{\psi}_{tilt}^{B}} \zeta\left(\boldsymbol{\psi}_{tilt}^{B}\right) = \max_{\boldsymbol{\psi}_{tilt}^{B}} \sum_{\forall s\in\mathcal{S}} \log_2\left(1+\gamma_s^b\left(\boldsymbol{\psi}_{tilt}^{B}\right)\right) \tag{11}$$

The points (CGs) in set $\mathcal{S}$ are shown in figure 3, where circles represent RSs i.e. points in set $\mathcal{R}$; and stars represent the CGs of users' geographical distribution in cells with no RS or with RS whose backhaul is not critical for the optimization process i.e. RS with $w_r < \frac{\sum_{\forall u\in\mathcal{U}_b^b} a_u}{\sum_{\forall u\in\mathcal{U}_b} a_u}$. Note that $|\mathcal{S}| << \left|\mathcal{U}\setminus\acute{\mathcal{U}} \cup \mathcal{R}\right|$. Thus, as highlighted in section III-B, for designing a SO solution, (11) is the required simplified manifestation of the original problem in (7).

D. Decomposition into Local Subproblem

As discussed in section III-B, for a distributed SO solution, after simplifying the original problem in (7) into (11) its decomposition into local subproblems is required to transform it from a large scale optimization problem to a small scale optimization problem. Such decomposition is common in SO systems in nature, as it is explained for the case study of flock of common cranes, in [17] and [18]. We refer to the same case study and more particularly to the result, discussed in the above references, according to which, for achieving the flock-wide objective of flying in V-formation, each crane merely relies on the observation of its immediate two neighbours, on its two sides. Thus, although cranes do not achieve the perfect V shape, they can still achieve up to 70% gain in group flight efficiency [17]. To exploit the same principle in our problem, we compromise on the global optimisation perspective and we propose the novel concept of *triplet* to enable a local problem decomposition. A *triplet* consists of three immediate neighbour cells as it is illustrated in the enlarged part of Figure 3. The key idea is that, as it happens for the cranes, each cell observes (tilts and CG locations) of its immediate two neighbours cells, when optimising its own tilts. In this way, tilts are optimised within each of the $N = \frac{B}{\hat{B}}$ triplets independently, where $\hat{B}$ (=3 in this case) is the size of the triplet, i.e. size of the local coordination group. As a result, the problem in (11) can be approximated as:

$$\max_{\boldsymbol{\psi}_{tilt}^{\hat{B}}} \hat{\zeta}\left(\boldsymbol{\psi}_{tilt}^{\hat{B}}\right) = \max_{\boldsymbol{\psi}_{tilt}^{\hat{B}}} \frac{1}{|\hat{\mathcal{B}}|} \sum_{\forall s\in\hat{\mathcal{S}}} \log_2(1+\hat{\gamma}_s^b) \tag{12}$$

where $\hat{\gamma}_s^b$ is the approximate SIR at point s (CG) that takes into account the observations from the only two other members of the triplet, and can be rewritten as:

$$\hat{\gamma}_s^b\left(\boldsymbol{\psi}_{tilt}^{\hat{B}}\right) = \frac{h_s^b 10^{\mu\left(\left(\psi_s^b-\psi_{tilt}^b\right)^2+c_s^b\right)}}{\sum_{\forall\check{b}\in\hat{\mathcal{B}}\setminus b}\left(h_s^{\check{b}} 10^{\mu\left(\left(\psi_s^{\check{b}}-\psi_{tilt}^{\check{b}}\right)^2+c_s^{\check{b}}\right)}\right)} \tag{13}$$

where b represents the antenna location of the cell which point s lies. $\hat{\mathcal{B}}$ represents a triplet, such that $\left|\hat{\mathcal{B}}\right| = \hat{B} = 3$. $\boldsymbol{\psi}_{tilt}^{\hat{B}}$ is the vector of tilt angles of the $\hat{B}$ sectors within the triplet. Use of triplet is further justified by the following propositions:

Proposition 1. *As β and the cell radius grows large, $\hat{\zeta}$ becomes a closer approximation of ζ.*

Proof: Proposition 1 can be easily proved by putting large values of β and d in (6) *and* (13). ■

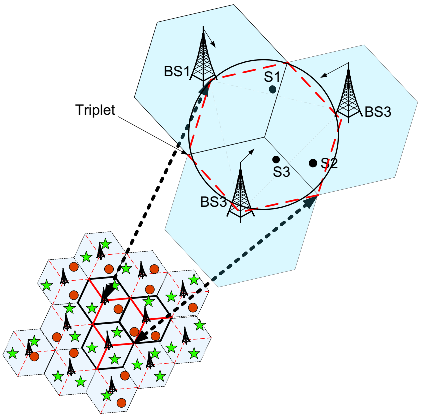

Fig. 3. Circles represent points in set $\mathcal{R}$ i.e. RS locations and stars represent points in set $\mathcal{V}$ i.e. focal points of user distributions in a cell determined through theorem (1) and its corollaries. Stars and circles together make set $\mathcal{S}$

Proposition 2. *If the SIR is given by $\hat{\gamma}_s^b$, the maximum aggregate throughput achieved in the system by optimizing the tilts within each triplet independently, is the same as the throughput achieved by optimizing the system-wide tilts. Mathematically, $\hat{\zeta}_{N,\max} = \hat{\zeta}_{max}$, where ζ_{max} is the maximum average SE that can be achieved by solving the optimisation problem in* (12) *and*

$$\hat{\zeta}_{N,\max} = \frac{1}{|\mathcal{N}|} \sum_{\forall n \in \mathcal{N}} \left\{ \max_{\boldsymbol{\psi}_{tilt}^{\mathcal{T}_n}} \frac{1}{|\mathcal{T}_n|} \sum_{\forall s \in \mathcal{S}_n} \log_2\left(1 + \hat{\gamma}_s^b\left(\psi_{tilt}^{T_n}\right)\right) \right\} \tag{14}$$

where $\mathcal{S}_n \subset \mathcal{S}$, $\mathcal{T}_n$ is the n^{th} triplet and $|\mathcal{S}_n| = |\mathcal{T}_n| = \mathcal{T}_n = 3, \forall n \in \mathcal{N}$, $\boldsymbol{\psi}_{\boldsymbol{tilt}}^{\boldsymbol{T_n}}$ is the vector of tilt angles of sectors within n^{th} triplet such that

$$\mathcal{S}_n \cap \mathcal{S}_{n'} = \Phi \text{ and } \mathcal{T}_n \cap \mathcal{T}_{n'} = \Phi \text{ }, \forall n \neq n' \text{ where } n, n' \in \mathcal{N} \tag{15}$$

$\mathcal{N}$ is set of all the triplets, such that $|\mathcal{N}| = \frac{|\mathcal{B}|}{|\mathcal{T}_n|}$=N is the total number of triplets in the system.

Proof: Since$|\mathcal{N}| \times |\mathcal{T}_n| = |\mathcal{N}| \times |\mathcal{S}_n| = |\mathcal{B}| = |\mathcal{S}|$ and in right hand side of (14) *all the terms are mutually exclusive hence the proposition.* ■

Each term in the summation in (14) is now a very small scale optimization subproblem over only three tilt angles within each triplet. Next we present a methodology to solve this subproblem.

E. Solution of the Local Subproblem

To enhance the SE on BS-RS, and BS-user links through optimisation of system wide antenna tilts, the following sub-problem needs to be solved for each of the N triplets locally and independently:

$$\max_{\psi_{tilt}^1, \psi_{tilt}^2, \psi_{tilt}^3} \hat{\zeta}\left(\psi_{tilt}^1, \psi_{tilt}^2, \psi_{tilt}^3\right) \tag{17}$$

subject to: $\qquad \psi_{tilt}^1, \psi_{tilt}^2, \psi_{tilt}^3 < \frac{\pi}{2}$

where $\hat{\zeta}$ is given by (16). We drop the subscript n to simplify the notation and since the following analysis is valid for any triplet. Notice that (17) is still a non convex optimization problem. However, compared to our original problem in (7), the problem in (17) is now a small scale optimisation problem, as the number of optimisation parameters is only three with limited range of $0° < \psi < 90°$. In most cases, while considering commercial tower heights and cell radii, the optimal tilt would lie in the range of $0° < \psi < 20°$. Since the search space of this problem is now reasonably small ($\approx 20 \times 20 \times 20 = 8000$), any of exhaustive search based evolutionary heuristics listed in [2] can be used to find the solution of (17). Alternatively, a solution can also be determined using a non linear optimization techniques that can tackle a non-convex optimisation objective. For example, since the objective function is twice differentiable and the constraint is also differentiable, an option could be to solve (17) through Sequential Quadratic Programming (SQP). To this end, the problem can be written in the standard form as:

$$\min_{\boldsymbol{\psi}} \quad -\hat{\zeta}\left(\boldsymbol{\psi}\right) \tag{18}$$

subject to: $\qquad g_j\left(\psi_j\right) < 0 \quad , j = 1, 2, 3$

where $\boldsymbol{\psi} = [\psi_1, \psi_2, \psi_3]$ and $g_j\left(\psi_j\right) = \psi_j - \frac{\pi}{2}$.

The Lagrangian of the problem in (18) is given by:

$$\mathcal{L}\left(\boldsymbol{\psi}\,, \boldsymbol{\lambda}\right) = \hat{\zeta}\left(\boldsymbol{\psi}\right) - \sum_{j=1}^{3} \lambda_{\mathbf{j}}(\psi_j - \frac{\pi}{2}) \tag{19}$$

If $\hat{\boldsymbol{H}}$ denotes the approximation of the Hessian matrix $\mathbf{H}$, then we can define a quadratic subproblem to be solved at the i^{th} iteration of SQP as follows:

$$\min_{\boldsymbol{w} \in \Re^J} \quad \frac{1}{2} \boldsymbol{w}^T \hat{\boldsymbol{H}} \left(\mathcal{L}\left(\boldsymbol{\psi}\,, \boldsymbol{\lambda}\right)\,\right)_i \boldsymbol{w} + \bigtriangledown \hat{\zeta}(\boldsymbol{\psi})_i \boldsymbol{w} \tag{20}$$

subject to: $\qquad w_j + \psi_{j_i} - \frac{\pi}{2} < 0 \quad j = 1, 2, 3$

At each iteration the value of $\hat{\boldsymbol{H}}$ can be updated using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) approximation method. Once the Hessian is known the problem in (20) is a quadratic programming problem that can be solved using standard methods such as the gradient projection [19].

Based on the above steps of the SQP, the problem in (17) can be solved within each triplet independently. The solution provides the optimal tilt angles to be maintained by each of the three cells in the triplet for given locations of CGs, within that triplet. The execution of these local solutions in each triplet results in the achievement of the system-wide objective in (11), which was a close manifestation of the original system wide objective in (7). Since the solution is distributed, i.e. executable in each triplet independently and autonomously, the near optimal tilt angles can always be maintained locally to maximise system-wide SE on the backhaul links, as well as on the coverage links, despite the impromptu deployment or removal of RSs. In the following we refer to the developed framework as SOT (Self-Organisation of Tilts).

IV. Performance Evaluation

Key modelling parameters used in system level performance evaluation are 3GPP compliant and are listed in Table I

$$\hat{\zeta} = \log_2\left(1+\left(\frac{h_1^1 10^{-1.2\mu\left((\psi_1^1-\psi_{tilt}^1)^2+c_1^1\right)}}{\left(h_1^2 10^{-1.2\mu\left((\psi_1^2-\psi_{tilt}^2)^2+c_1^2\right)}\right)+\left(h_1^3 10^{-1.2\mu\left((\psi_1^3-\psi_{tilt}^3)^2+c_1^3\right)}\right)}\right)\right)+$$
$$\log_2\left(1+\left(\frac{h_2^2 10^{-1.2\mu\left((\psi_2^2-\psi_{tilt}^2)^2+c_2^2\right)}}{\left(h_2^1 10^{-1.2\mu\left((\psi_2^1-\psi_{tilt}^1)^2+c_2^1\right)}\right)+\left(h_2^3 10^{-1.2\mu\left((\psi_2^3-\psi_{tilt}^3)^2+c_2^3\right)}\right)}\right)\right)+$$
$$\log_2\left(1+\left(\frac{h_3^3 10^{-1.2\mu\left((\psi_3^3-\psi_{tilt}^2)^2+c_3^3\right)}}{\left(h_3^1 10^{-1.2\mu\left((\psi_3^1-\psi_{tilt}^1)^2+c_3^1\right)}\right)+\left(h_3^2 10^{-1.2\mu\left((\psi_3^2-\psi_{tilt}^2)^2+c_3^2\right)}\right)}\right)\right) \quad (16)$$

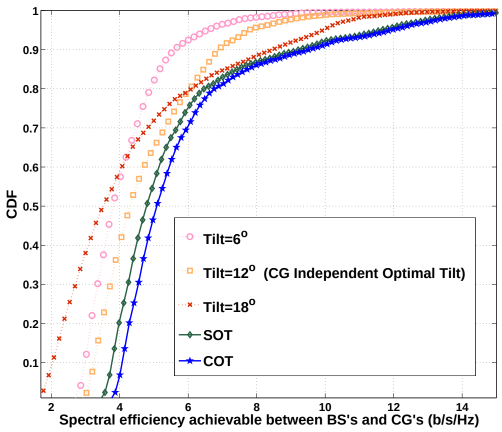

Fig. 4. SOT is compared with fixed tilting as well as Centralized Optimization of Tilts (COT). COT solution is obtained by solving (11) via brute force.

TABLE I
3GPP COMPLIANT SYSTEM LEVEL SIMULATION PARAMETERS

Parameters	Values
System topology	19 BS with 3 sector/cells per BS
BS Transmission Power	46 dBm
BS Inter site distance	500 meters
BS height	32 meters
RS height	5m
RS Type	Capacity Extension i.e. $w_r = 1, \forall r \in R$
User height	1.5 meters
User activity levels	$a_u = 1, \forall u \in U$
Network Topology Type	Homogenous, $w_s = 1, \forall s \in S$
User antenna	5 dB (Omni directional)
RS antenna	7 dB (Omni directional)
horizontal beamwidth, B_h	70^0
vertical beamwidth, B_v	10^0
vertical Gain Weight , λ_v	0.5
vertical Gain Weight , λ_h	0.5
maximum gain, G_{max}	14 dB
maximum attenuation, A_{max}	25 dB
Frequency	2 GHz
Pathloss model	Urban, Scenario 1 [20]

A. Comparing SOT with a Centralized Optimal Solution

In this section we compare SOT with fixed optimal tilting as wells with Centralized Optimization of Tilts (COT). The COT solution is obtained by solving (11) through brute force for $7 \times 3 = 21$ cells. Due to the computational time constraint for COT, only a tilt range of $6^0 - 18^0$ is considered with a resolution of 2^o. The rationale behind selecting this range is that it is centered around fixed optimal tilt 12^0. i.e. since incase of perfectly uniform user distribution geometical centroid of the cell becomes its CG, the fixed optimal tilt for a given BS height of 32m, user height of 1.5m and intersite distance of 500m (see Table I) is 12^o, i.e. $\arctan((32-1.5)/(\frac{500}{2cos(30)2}) \approx 12^o$. Thus 6^{21} evaluations of the objective function in (11) are traversed to find the optimal solution. On a regular desktop computer (2.8 GHz processor, 8GB RAM) it took well over 8 hours. For fair comparison, SOT is also implemented under the same set up of tilt range, resolution and number of cells in the system.

Figure 4 plots the CDF of spectral efficiency achievable on links assumed between CGs and BS, with SOT and COT. Note that albeit relying on local information only, SOT's performance is considerably close to COT. As expected, being globally optimal, COT does outperform SOT slightly. However, note that from a real world implementation point of view COT is difficult to implement not only because of the tremendous computation effort required but also due to the global signalling needed for its implementation. In figure 4, the CDFs with fixed optimal tilting and other typical fixed tilting values are also plotted for SOT's comparison with fixed tilting that is often empirically set in commercial cellular systems. It can be noted that SOT outperforms all fixed tilting schemes including the fixed optimal tilt of 12^0.

B. System Level Simulation Results

Our system level simulator models an OFDMA based generic cellular system where half of the cells contain randomly located RS and the other half, selected randomly, do not have RSs and are served by the BS only. Due to space limitation, we present results for capacity enhancing RS only, as only in this case, does the backhaul optimization become significant. To model the capacity enhancing RS scenario, we assume that in the cells with RSs, 80% of the users in that cell are concentrated within 200m radius of the RS. In cells without RSs, users are randomly distributed across the cell. The simulator, is snapshot based and results reported are averaged over 10 snapshots of user and RS locations and tilt settings obtained via SOT for these user and RS distributions. Again comparison with prior works on heuristic

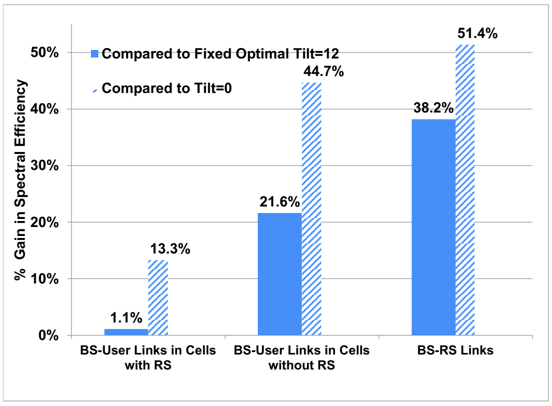

Fig. 5. Average spectral efficiency with SOT and with fixed tilts on BS-RS and BS-user links

based dynamic tilting schemes is omitted because of the reasons explained above. Instead, for the sake of reproducible performance evaluation, we compare the performance of SOT against a range of typical fixed antenna tilts including the fixed optimal tilt i.e. $0^o, 6^o, 12^o, 18^o$. Performance is evaluated for both BS-RS access links as well as BS-user coverage links. Figure 5 that plots the percentage gain in average spectral efficiency SOT yielded when compared to the fixed optimal tilt of 12^0 and no tilting at all. It can be observed that for the cells with RS, BS-user links of the 20% users that are not explicitly considered by SOT while determining tilt, no significant gain is achieved compared to fixed optimal tilt, as expected. However, for rest of the users, as well as, RSs that are considered in determining the CGs, SOT yields very substantial gains compared to fixed tilting.

V. Conclusions

A novel analytical framework for self-organisation of BS tilts (referred in the text as SOT) is developed by exploiting a unique system level perspective to jointly self-optimise the spectral efficiency on the BS-RS backhaul links and coverage links in a relay enhanced cellular system. Results show that a gain of 10-50% in spectral efficiency compared to typical fixed optimal tilting can be obtained with SOT depending on system topology and user demography. SOT yields this gain as it calculates and then dynamically adapts cell specific optimal tilt values by taking into account users and RSs locations and activity levels. In this paper we demonstrated the SOT's gain mainly in context of a hexagonal grid model only, for tractability and brevity reasons. However, SOT is implementable in a real heterogeneous network as long as the network topology allows decomposition into local non-overlapping cluster of cells (e.g. quartet, quintet, sextuplet) with the same property as a triplet i.e. a set of most interfering cells that can be repeated to cover the whole network without overlap. The weight factors incorporated into the framework while calculating CGs can actually be used to take into account other types of heterogeneity such as cell sizes, sector spreads and azimuth angle biases, in addition to the user profiling and RS types. Though the exact gain of SOT will vary depending on actual system parameters and topology, as pointed out in the results section, the key advantage of SOT is that it is practically and cost-efficiently implementable in distributed and scalable fashion in both existing and emerging cellular systems.

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