Continuous Time Markov Chain Based Reliability Analysis for Future Cellular Networks

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Abstract—**It is anticipated that the future cellular networks will consist of an ultra-dense deployment of complex heterogeneous Base Stations (BSs). Consequently, Self-Organizing Networks (SON) features are considered to be inevitable for efficient and reliable management of such a complex network. Given their unfathomable complexity, cellular networks are inherently prone to partial or complete cell outages due to hardware and/or software failures and parameter misconfiguration caused by human error, multivendor incompatibility or operational drift. Forthcoming cellular networks, vis-a-vis 5G are susceptible to even higher cell outage rates due to their higher parametric complexity and also due to potential conflicts among multiple SON functions. These realities pose a major challenge for reliable operation of future ultra-dense cellular networks in cost effective manner. In this paper, we present a stochastic analytical model to analyze the effects of arrival of faults in a cellular network. We exploit Continuous Time Markov Chain (CTMC) with exponential distribution for failures and recovery times to model the reliability behavior of a BS. We leverage the developed model and subsequent analysis to propose an adaptive fault predictive framework. The proposed fault prediction framework can adapt the CTMC model by dynamically learning from past database of failures, and hence can reduce network recovery time thereby improving its reliability. Numerical results from three case studies, representing different types of network, are evaluated to demonstrate the applicability of the proposed analytical model.**

Keywords—Self Organizing Networks (SON); SON Conflicts; Continuous Time Markov Chain (CTMC); Reliability; 5G Networks; Ultra Dense Networks

I. INTRODUCTION

The envisioned fifth Generation (5G) Cellular Networks are expected to achieve 1000 times capacity gain mainly through extreme network densification (Fig. 1) [1]. Moreover, with each successive generation of cellular networks, complexity of BS has continued to increase i.e. a typical 2G cell has 500 parameters to optimally configure and maintain; 3G cell has 1000; and 4G has roughly 1500 parameters. Without intervening measures, same complexity growth trend is expected for 5G [1]. To efficiently manage such an ultra-dense, complex, heterogeneous cellular networks, the paradigm of SON has recently been
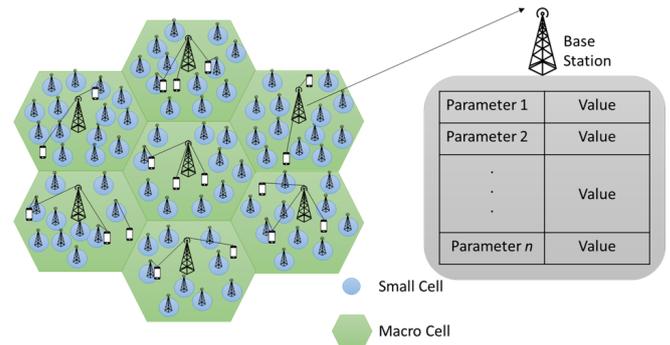

Fig. 1. Ultra Dense Heterogeneous Complex Cellular Network

investigated heavily to automate network configuration and management tasks [2]. Realizing the importance of SON as a key enabler for future cellular networks, a number of SON use cases have already been standardized by 3GPP [3]. There is a general consensus that SON will not be a luxury but a necessity in 5G networks [4]. SON functions, classified as (1) Self-Configuring, (2) Self-Optimizing and (3) Self-Healing, operate by reconfiguring a number of network parameters. For a thorough review of state of the art SON function see [5].

Cellular networks are inherently subject to cell outages caused by either BS hardware and/or software malfunctions or misconfiguration of several hundred cell parameters during routine network operation. Forthcoming cellular networks are susceptible to even higher cell outages rates as the multiple SON functions may be subjected to large number of potential conflicts when operated concurrently in a system. Given the parametric overlap as well as coupling among the objectives of different SON functions, it has been shown in [6] that large number of conflicts are possible among SON functions if no self-coordination mechanism is employed. At times, such conflicts can actually degrade networks performance instead of improving it. For example capacity and coverage optimization SON function might try to improve coverage by increasing transmission power. This may conflict with energy efficiency SON. In summary, as identified in [6], in an uncoordinated SON, a variety of conflicts may occur when: 1) Two or more SON functions try to modify the same network configuration parameter 2) A SON function is triggered by an input parameter whose value

is dependent upon some other network parameters 3) There is a change in network conditions by impromptu addition or removal of relay, eNB or Home eNB (HeNB) 4) Different SON function actions try to alter the same KPI of a cell, while adjusting different network configuration parameters 5) A SON function computes new parameter configuration values based on outdated measurements 6) There is a logical dependency among the objectives of SON functions.

The potential failures occurring due to hardware/software malfunctioning, multi-vendor incompatibility or SON conflicts ultimately affect the coverage and performance reliability of the network. BS can be susceptible to complete outage due to critical failures or can exhibit degraded performance in case of trivial failures. The reliability analysis of future cellular network's BS is of paramount importance for network operators since it directly effects the Quality of Service and user experience. A quantitative analysis of SON reliability can also give vendors a better insight into the various reliability considerations in SON. It can also help improve operator's confidence on SON which has been major bottleneck in SON penetration despite of the enormous financial and technical gain SON can offer. Despite the great significance of the topic, so far very few studies have focused on the reliability and survivability analysis of cellular networks in general and SON enabled cellular networks in particular. Dharmaraja et al. [7] developed analytical model for reliability and survivability quantification of a UMTS architecture network. Xie et al. [8] modeled and analyzed the survivability of an infrastructure based wireless network under disaster propagation. Tipper et al. [9] did simulation based survivability analysis of a mobile network. Guida et al. [10] evaluated performance analysis of IP Multimedia Subsystem (IMS) core network signaling servers. However, unlike the previous works on cellular network reliability that mostly focus on the structural aspects of cellular networks and overlook the network behavioral aspects that can cause complete or partial failures , our work is more focused towards developing a generic analytical model encompassing diverse faults cases like software/hardware failures or SON conflict attributed misconfigurations. This approach gives the flexibility to incorporate variety of failure scenarios into the model. To the best of our knowledge a study that analyzes the probabilistic reliability behavior of the SON enabled emerging cellular networks including 5G by considering the failure probability of BSs using CTMC does not exist in open literature. This paper is first attempt in that direction. Rest of this paper is organized as follows: Section II presents the reliability behavior model of the BS. In Section III, we have performed Transient analysis along with the computation of performance metrics. Numerical results are presented in Section IV while the utility of the developed model is presented in Section V. In Section VI, key conclusions and Future works are discussed.

II. MODEL DEVELOPMENT

To analyze and evaluate the reliability behavior of a cellular network, a quantitative model for a cell (BS) failure is needed. In real-world cases, most of the node failure and repair times follow time-dependent failure rate distributions such as Weibull, Pareto and lognormal [11]. However, in most cases, analytical models with general (non-exponential) distributions are not mathematically tractable. Therefore, phase-type distribution, which is convolution of many exponential phases is used for approximating many general distributions and is used to construct the mathematically solvable analytical models [7], for component reliability analysis [12]. Since exponential distribution is a particular case of phase type distribution, hardware and software faults are commonly modeled as exponential distribution. Therefore in this paper, we consider that the time to transit from a system state to another due to failures and recovery also follows an exponential distribution. This assumption is also supported by the fact that the exponential random variable is the only continuous random variable with Markov property. Building on this assumption we construct an analytically tractable CTMC model for reliability analysis of SON enabled BS. Fig. 2 shows the state transition diagram of the CTMC model for the probabilistic reliability behavior of the BS.

Let $X(t)$ with finite State Space S = {1, 2, 3} denote the state of the BS at time t wherein:

- $X(t)$ = 1 if BS is in healthy state at time t with all parameters configured with optimal value.

- $X(t)$ = 2 if BS is in sub-optimal state at time t with one or more of the parameters misconfigured. In this state, cell continue to operate but its performance degrades below a typical level of performance.

- $X(t)$ = 3 if BS is in complete outage at time t.

It is assumed that time for failure is exponentially distributed. Since the rate of arrival of failures is temporarily independent, it can be modelled using Poisson distribution. We classify failures into (1) Trivial failures characterized by arrival rate λ_t. Trivial failures are the failures which do not cause complete outage but drive the network state from optimal to Sub-optimal state. (2) Critical failures characterized by arrival rate λ_c. Critical failures lead to complete outage of the cell. Therefore, trivial failures can only occur when network is in optimal state while critical failures can occur in state 1 or state 2. Each BS is assumed to be equipped with recovery module powered by self-coordination framework such as proposed in [6]. This module reconfigures all configuration parameters back to their original optimal values once the misconfiguration is detected and diagnosed. Moreover it has the capability to reset the BS software or switch over to the secondary backup hardware board if failure has stemmed from hardware/software related issues. The time to move the
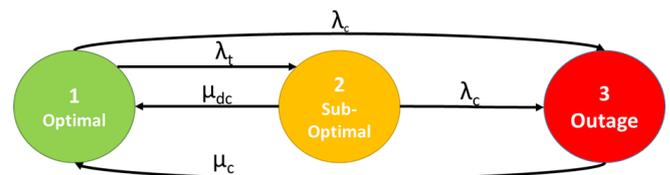

Fig. 2. State Transition Diagram for SON enabled BS

network from sub-optimal state back to optimal state is assumed to be exponentially distributed with mean value $1/\mu_{dc}$. This includes the time for cell anomaly detection, diagnosis and compensation [13, 14]. Similarly the time period to recover from complete outage is exponentially distributed with mean value $1/\mu_c$ which generally involves time for compensation only. Furthermore, the failure or repair transition is only determined by the current state and not on the path to the current state. With these assumptions, the transient process $X(t)$ can be mathematically modeled as a temporally homogeneous CTMC on the state space S. For each time $t > 0$, the probability that the BS is in state j is given by:

$$p_j(t) = \Pr\{X(t) = j\}, j \in S \qquad (1)$$

Let $p(t) = [p_1(t), p_2(t), p_3(t)]$ denote row vector of transient state probabilities of $X(t)$. The generator matrix Q and rate matrix R for this CTMC $X(t)$ are given as:

$$Q = \begin{bmatrix} -\lambda_t - \lambda c & \lambda_t & \lambda_c \\ \mu_{dc} & -\mu_{dc} - \lambda_c & \lambda_c \\ \mu_c & 0 & -\mu_c \end{bmatrix} \qquad (2)$$

$$R = \begin{bmatrix} 0 & \lambda_t & \lambda_c \\ \mu_{dc} & 0 & \lambda_c \\ \mu_c & 0 & 0 \end{bmatrix} \qquad (3)$$

III. ANALYSIS

In this section we perform Transient Analysis followed by the computation of performance metrics.

A. Transient Analysis

Using generator matrix Q, the dynamic behavior of the CTMC can be described by the Kolmogorov differential equation in the matrix form:

$$\frac{dP(t)}{dt} = P(t)\, Q \qquad (4)$$

Then the transient state probability vector can be obtained as:

$$P(t) = P(0)e^{Qt} \qquad (5)$$

Where $P(0)$ is initial probability vector. The common methods to obtain the transient probability vector $P(t)$ includes matrix exponential approach [15] and uniformization [16]. In this paper, we resort to uniformization method for the analysis because of its higher accuracy and efficient computation due to which it is the method of choice for typical problems similar to one under consideration in this paper [17].

Let q_{ii} be the diagonal element of Q and I be the unit matrix, then the transient state probability vector can be obtained as follows:

$$P(t) = \sum_{k=0}^{\infty} e^{-\beta t}\, \frac{(\beta t)^k}{k!}\, \widehat{P}^k \qquad (6)$$

Where $\beta \geq \max_i |q_{ii}|$ is uniform rate parameter and $\widehat{P}$ is Probability Transition Matrix given as:

$$\widehat{P} = I + Q/\beta \qquad (7)$$

When we truncate the summation in (6) from infinity to some large number M, the resulting controllable accuracy error can be computed as:

$$\epsilon = 1 - \sum_{k=0}^{M} e^{-\beta t}\, \frac{(\beta t)^k}{k!} \qquad (8)$$

B. Performance Metrics

Based on the uniformization method, three performance metrics to quantify the reliability of network are computed as follows:

1) Occupancy Time:

The expected length of time the BS spends in each of the states {Optimal, Suboptimal, Outage} during a given interval of time can be determined using occupancy time of the CTMC. Let $\Psi_{i,j}(T)$ be the expected amount of time the CTMC spends in state j during the interval $[0, T]$, starting in state i and $p_{i,j}(t)$ be the element of the transition probability matrix $\widehat{P}$. The quantity $\Psi_{i,j}(T)$ is called the occupancy time of state j . until time T starting from state i given as:

$$\Psi_{i,j}(T) = \int_0^T p_{i,j}(t)\, dt \qquad (9)$$

and in matrix form:

$$\Psi(T) = \begin{bmatrix} \Psi_{1,1} & \Psi_{1,2} & \Psi_{1,3} \\ \Psi_{2,1} & \Psi_{2,2} & \Psi_{2,3} \\ \Psi_{3,1} & \Psi_{3,2} & \Psi_{3,3} \end{bmatrix} \qquad (10)$$

2) First Passage Time:

The expected value of the random time at which BS passes into each of the states {Optimal, Suboptimal, Outage} for the first time can be calculated using first passage times of the CTMC. The first-passage time ζ_j into state j starting from state i is defined to be:

$$\zeta_j = E(T|X(0) = i) \qquad (11)$$

where

$$T = \min\{t \geq 0 : X(t) = j\} \qquad (12)$$

and E is the expected value. The first passage times for a CTMC with a State Space S satisfy the following relation [16]:

$$r_i \zeta_i = 1 + \sum_{j=1}^{N-1} r_{i,j} \zeta_j, \qquad 1 \leq i \leq N - 1 \qquad (13)$$

Where $i, j \in S$ and $r_i = \sum_{j=1}^{N} r_{i,j}$, $R = [r_{i,j}]$.

Therefore in our model, first passage time for state 2 will be:

$$(\lambda_t + \lambda_c)\zeta_1 = 1 + \lambda_c \zeta_3 \qquad (14)$$
$$\mu_c \zeta_3 = 1 + \mu_c \zeta_1 \qquad (15)$$

By solving (14) and (15) we get:

$$\zeta_{3\to2} = \left(\frac{(\lambda_t + \lambda_c) + \mu_c}{((\lambda_t + \lambda_c) * \mu_c) - \lambda_c\mu_c} \right) \tag{16}$$

$$\zeta_{1\to2} = \left(\frac{\mu_c\zeta_{3\to2} - 1}{\mu_c} \right) \tag{17}$$

Where $\zeta_{3\to2}$ and $\zeta_{1\to2}$ are the first passage times to state 2 starting from states 3 and 1 respectively. First Passage time for State 3 will be:

$$(\lambda_t + \lambda_c)\zeta_1 = 1 + \lambda_t\zeta_2 \tag{18}$$

$$(\mu_{dc} + \lambda_c)\zeta_2 = 1 + \mu_{dc}\zeta_1 \tag{19}$$

By solving (18) and (19) we get:

$$\zeta_{1\to3} = \frac{\mu_{dc} + \lambda_c + \lambda_t}{(\lambda_t + \lambda_c)(\mu_{dc} + \lambda_c) - \lambda_t\mu_{dc}} \tag{20}$$

$$\zeta_{2\to3} = \frac{1 + \mu_{dc}\zeta_{1\to3}}{(\mu_{dc} + \lambda_c)} \tag{21}$$

Where $\zeta_{1\to3}$ and $\zeta_{2\to3}$ are the first passage times to state 3 starting from states 1 and 2 respectively.

3) Steady State Distribution:

In order to analyze the long term behavior of the network, we evaluate the limiting distribution of this CTMC. The limiting or steady state distribution ψ is defined as:

$$\psi = [\psi_1, \psi_2, \psi_3] \tag{22}$$

Where

$$\psi_j = \lim_{t\to\infty} \Pr(X(t) = j) \tag{23}$$

For a CTMC with rate matrix $R = [r_{i,j}]$, it is calculated as:

$$\psi_j r_j = \sum_{i=1}^{N} \psi_i r_{i,j} \tag{24}$$

and

$$\sum_{i=1}^{N} \psi_i = 1 \tag{25}$$

Therefore for our model we determine $[\psi_1 \; \psi_2 \; \psi_3]$ by solving:

$$A\psi = B \tag{27}$$

where

$$A = \begin{bmatrix} \lambda_t + \lambda_c & -\mu_{dc} & -\mu_c \\ \lambda_t & -(\mu_{dc} + \lambda_c) & 0 \\ \lambda_c & 0 & -\mu_c \\ 1 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

IV. NUMERICAL RESULTS

For numerical results, we considered three case studies with parameter settings as shown in Table I. In Case Study I, trivial failures are assumed to occur with mean value of 8 hours in relation to the traffic pattern shifts during morning, evening and night times which might trigger a number of SON functions. Probability of occurrence of critical failures is assumed to be 10 time less than those of trivial failures. Normally cell outage detection is not a straight forward task

TABLE I. MODEL PARAMETERS FOR CASE STUDIES

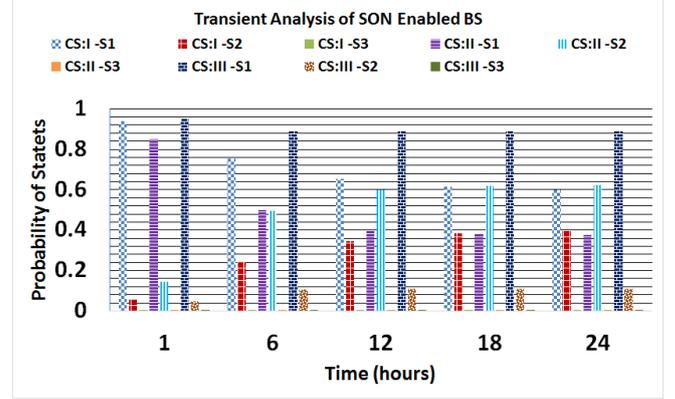| Parameter	Case Study: I	Case Study: II	Case Study: III
λ_t hour^{-1}	1/8	1/3	1/8
λ_c hour^{-1}	1/80	1/30	1/80
μ_{dc} hour^{-1}	1/6	1/6	1
μ_c hour^{-1}	12	12	12
Error	0.00001	0.00001	0.00001

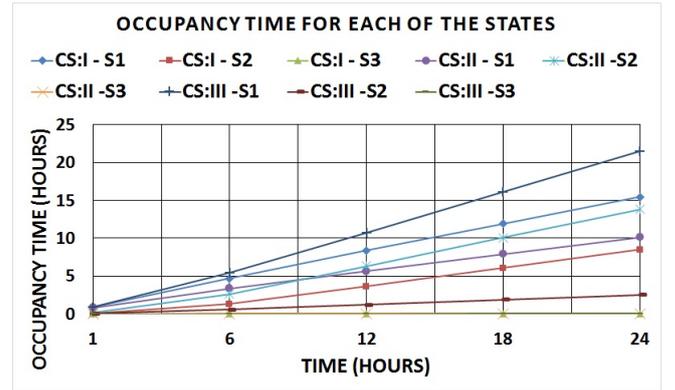Fig. 3. Transient Analysis of SON enabled BS for three case studies

Fig. 4. Occupancy Time of SON enabled BS for three case studies

and it may take several hours for detection, diagnosis and compensation of outages. Therefore we considered μ_{dc} to be exponentially distributed with mean value of 6 hours. In case study I, compensation is assumed to have mean value of 5 minutes and also has exponential distribution. This small recovery time makes sense only when it is assumed that the SON self-healing functions involving automated diagnosis, such as proposed in [13, 14] will be invoked for the recovery process. Otherwise, a recovery time can be significantly large. In case study II, we increased the arrival rate of misconfigurations (trivial faults) from one per eight hour to one per three hours. Arrival time for critical faults is assumed to be one per 30 hours. Case study II is meant or represent densely deployed cells where SON functions may need to be activated and deactivated more frequently, for example ultra-dense mmWave based deployment in 5G. In case study III, all parameters are assumed to be same as case study I except detection and compensation time that in case

study I is assumed to be exponentially distributed with a mean value of 1 hour.

Transient Analysis of the three case studies is shown in Fig. 3. For case study I, the probability of BS to be in optimal healthy state is around 95% after 1 hour and gradually decreases to around 60% after 24 hours period. There is a very low probability of the BS to be in outage state as critical failures rate is too small in our assumed model. For case study II, the probability of the network to be in sub-optimal state is 15% after 1 hour and it gradually increases to 62% after 24 hours since rate of arrival of trivial failures is high in case study II. In case study III, the probability of the BS to be in optimal state is around 88% after 24 hours. This indicates that decreased detection and compensation time has a profound effect on the network performance reliability. Therefore failures detection, diagnosis and compensation time should be as small as possible for achieving maximum performance. This calls for need for more agile self-healing methods in emerging cellular networks where increased complexity might cause higher fault arrival rate. The self-healing methods proposed in recent studies such as [13, 14, 18] are good candidates to overcome this problem. The occupancy time for the three case studies is show in in Fig. 4. For case study I and III, the network remains in optimal state most of the time as compared to case study II in which sub-optimal time gradually increases with the passage of time. This is direct result of the higher rate of arrival for trivial faults in case study II. The first passage times into state 2 and 3 is shown in Fig. 5. The first passage time for the three case studies depends upon the mean arrival rate of trivial as well as critical failures so values of λ_t and λ_c both determine when a cell first experiences degradation and complete outage. As expected, the time to first experience sub-optimal performance is very small as compared to complete outage. First passage time is small in case study II as compared to the other two case studies due to higher rate of arrival of faults in case study II, compared to other two case studies. The limiting or steady state distribution is given in Fig. 6. In the long run, a cell remains 58.3% and 88.9% of the time in optimal state for the case studies I and III respectively. However, for the case study II, it stays only 36.17% of the time in optimal state (63.77% in sub-optimal state) due to higher rate of trivial failures. The BS stays negligibly small amount of time in state 3 as critical failure rate is very small in our study.
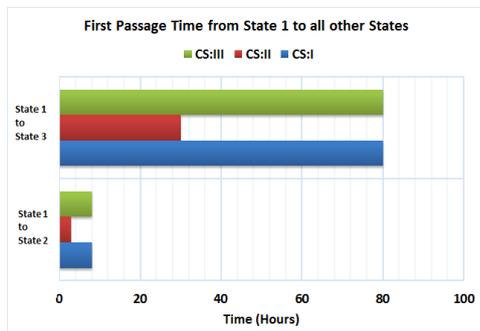

Fig. 5. First Passage Time of SON enabled BS for three Case Studies (CS)
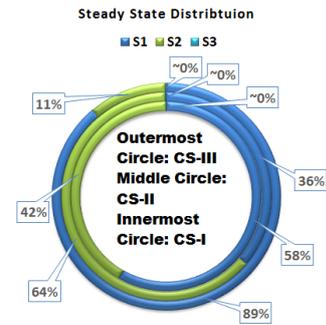

Fig. 6. Limiting (Steady State) Distribution of SON enabled BS for three case studies

V. UTILITY OF THE DEVELOPED MODEL: FAULT PREDICTION FRAMEWORK (FPF)

Utilizing the analytical model developed in previous section, we propose Fault Predictive Framework (FPF) which predicts the occurrences of faults based upon the past database of failures (Fig. 7). Historical data of past failures and misconfigurations of BS network parameters that occur routinely during operation of a cellular network can be utilized to estimate the λ_t, λ_c, μ_{dc} and μ_c parameters using standard machine learning tools. These estimated mean values then can be plugged into CTMC model and Q and R matrices can then be updated dynamically. The fitting of data to phase-type distributions has been covered by various research studies, such as in [19]. Based on updated Q and R matrices, Transient and Steady state analysis can then be run to compute new values for expected time for the first occurrence of fault, occupancy time and steady distribution. The difference between the predicted and actual values can be used to retrain the CTMC model parameters. In some of the cases, cell degradation is difficult to detect [20] e.g. in case of sleeping cells where no alarms are raised. In those cases, cell outage/degradation detection requires expensive site visits or drive testing that may take hours or days for the sub-optimal behavior to be detected. In majority of the cases, excessive customer complaints indicate the occurrence of faulty behavior of a cell. This results in significant reduction in quality of service and capacity. The probability of a cell to be sub-optimal at a given time period can be calculated by the proposed framework and can be exploited to minimize the degradation time. Once the predicted fault occurrence time is
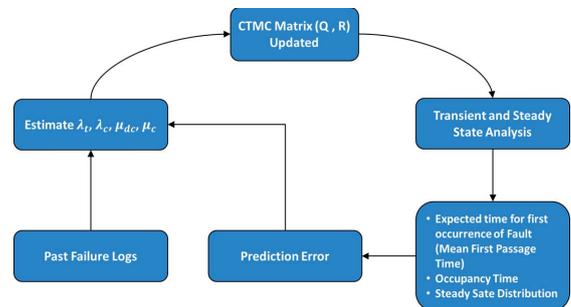

Fig. 7. Fault Predictiction Framework (FPF)

near, prioritizing verification of each of the BS element or the configuration parameters can be initiated. Similarly, Occupancy time of the BS or Steady State distribution can be used as a KPI for cell performance. If the calculated values suggest that the time period which the cell will spend in sub-optimal or outage state is above some threshold value than that cell can be prioritized accordingly in the optimization process. The proposed framework can also aid in the diagnosis of the faults as this is one of the most difficult tasks faced by the BS subsystem engineers. If some record is maintained for the time interval of occurrence of fault and the corresponding root cause of that fault, as the expected suboptimal behavior or outage time approaches, the diagnosis should start right from the root cause already recorded in the table. This can result in significant reduction in diagnosis time and compensation time. The CTMC model and associated FPF framework presented in this paper thus can significantly improve reliability of network and provide enhanced user experience as expected from 5G.

VI. CONCLUSION & FUTURE WORK

In this paper, we presented a stochastic analytical model to analyze the effects of arrival of faults on the reliability behavior of a cellular network. Assuming exponential distributions for failures and recovery, a reliability model is developed using CTMC process. The proposed model, unlike previous studies on network reliability is not limited to structural aspects of BSs, and takes into account diverse potential fault scenarios and is capable to predict the expected time of first occurrence of the fault and long term reliability behavior of the BS. This model can adapt itself dynamically by learning from past database of network failures. Three different scenarios have been analyzed in terms of transient analysis, occupancy time, first passage time and the steady state distribution. As per the numerical results, mean arrival rate of trivial failures has profound effect on the reliability behavior of the cellular network. Another key finding is that, substantial gain in network reliability can be achieved by reducing BS's fault detection and recovery time, which strongly advocate the need for agile self-healing SON functions.

As for future work, the proposed model will be extended with non-exponential distribution for failures and recovery times. Moreover, methods will be developed to efficiently estimate CTMC model parameters by learning from the past failure logs collected from real network.

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